LINEAR-PHASE FIR DIGITAL FILTER DESIGN
BY
WINDOWS METHOD.
EXAMPLES

Exercise 3.

1. Summary of Important Expressions

Table 1. FIR Linear Time - Invariant System Description: A Review of Basic Expressions

1.	Time – domain description	$y(n)=\sum_{k=0}^{M-1} h(k)x(n-k)$
2.	Frequency – domain description	$H(e^{j\omega})=\sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k} = \sum_{k=0}^{M-1} h(k)e^{-j\omega k}$
3.	Impulse response	$h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}\,d\omega$

Table 1.2. Some Commonly Used Windows for FIR Filter Design

	Window Type	**Window Functions,** $w(n)$ **,** $-M \le n \le M$ **,** $M=\frac{N-1}{2}$**,** $\lvert w(n)\rvert = 0 \;\; for \;\; n>M$
1.	Rectangular	$w(n)=1$
2.	Bartlett	$w(n)=1-\frac{\lvert n\rvert}{M+1}$
3.	Hann	$w(n)=\frac{1}{2}\left[1+\cos\frac{2\pi n}{2M+1}\right]$
4.	Hamming	$w(n)=0.54+0.46\cos\frac{2\pi n}{2M+1}$
5.	Blackmann	$w(n)=0.42+0.5\cos\frac{2\pi n}{2M+1}+0.08\cos\frac{4\pi n}{2M+1}$
6.	Kaiser (adjustable window) parameter: α	$w(n)=\frac{I_0\left(\alpha\sqrt{1-\left(\frac{n}{M}\right)^2}\right)}{I_0(\alpha)}$ $\quad I_0(x)=1+\sum_{r=1}^{\infty}\left(\frac{(x/2)^r}{r!}\right)^2$

Comments on Kaiser Window: $I_0(x)$ is the modified zero-th-order Bessel function of the first kind. For most practical applications, about 20 terms in the above summation are sufficient to arrive at reasonably accurate values of $w(n)$.

Table 1.3. Frequency Responses of Some Linear Time-Invariant Systems

	System	Frequency Response
1.	Differentiator	$H\left(e^{j\omega}\right)=\dfrac{j\omega}{T}, \quad -\pi \le \omega \le \pi .$
2.	Hilbert Transformer	$H(j\omega)=\begin{cases} -j & \omega>0 \\ 0 & \omega=0 \\ j & \omega<0 \end{cases}, \quad -\pi \le \omega \le \pi$

Example 1.1.
Design a band-pass filter with pass-band cut off frequencies $f_1 = 20\,kHz$ and $f_2 = 40\,kHz$ of the order $N = 11$. Frequency sampling is $f_S = 160\,kHz$. It is desired to apply rectangular and Bartlett window at the design.

Example 1.2.
By the impulse response truncation method (by the windowing method at rectangular window application) design a Hilbert transformer of the order $N = 11$.

Example 1.3.
By the windowing method at Hann window application design a differentiator of the order $N = 11$.

Example 1.4.
Design a stop-band filter with pass-band cut off frequencies $f_1 = 20\,kHz$ and $f_2 = 40\,kHz$ of the order $N = 11$. Frequency sampling is $f_S = 160\,kHz$. It is desired to apply rectangular and Bartlett window at the design.